Blue or bluer?

Suppose a rocket flies at high speed toward a stationary monochrome light source and carries equipment to measure its frequency. The light will be blueshifted because the rocket is moving toward the light. In addition special relativity, SR, predicts the time rate of the atoms in the moving rocket will be reduced, so the frequency of the light recorded will be slightly higher, i.e. it will be bluer for an observer on the moving rocket.

According to SR though the rocket can be regarded as at rest whilst the light source moves at high speed toward the rocket. It is now the time rates of the light's emitting atoms that are reduced, so this slightly reduces the frequency of the blue shifted light arriving at the "stationary" rocket, i.e. the light is not as blue as predicted for an observer on the moving rocket.

So the frequencies predicted by observers in different frames are different. When the rocket slows down to be at rest with the light source which prediction will be wrong?

(This involves two different predictions about a single physical measurement, not two measurements. This differs from the following two cases. Firstly, an observer is at rest with a clock and predicts that a measurement will show it is not time dilated. A moving observer predicts that the first observer is correct – the clock and observer having zero relative speed. So here two compatible predictions are made about a single measurement. Secondly, two observers in relative motion predict they will measure each other's clock to be time dilated. In this case there are two predictions about two different measurements.

An experiment can obviously invalidate a theory by showing its prediction is incompatible with a measurement. Although a thought experiment cannot produce a measurement, it can still invalidate a theory by showing it makes incompatible predictions about a single physical measurement as viewed in a single frame.)